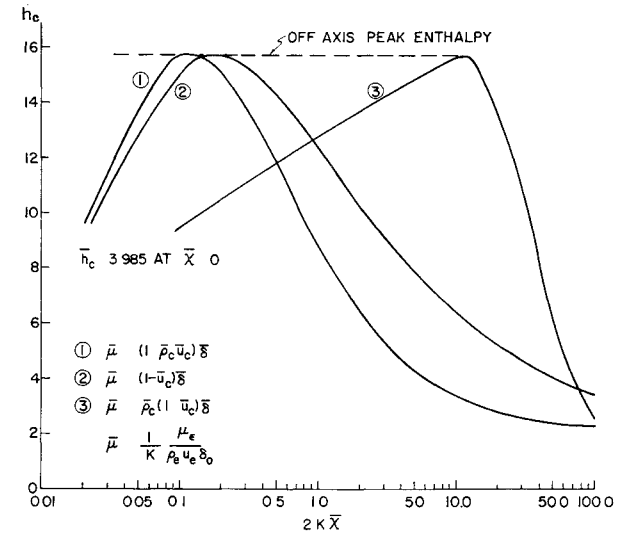

Fig 2 Variations of axis enthalpy; slender cone, altitude = 150,000 ft, M_∞ = 19 52

for the three different forms of the eddy viscosity coefficients For case 3, this distance is $(2K\bar{x})\bar{n}_{cm\ x} = 10$, which is two orders of magnitude higher than case 1, where $(2K\bar{x})\bar{n}_{cm\ x} = 0\,1$, whereas in case 2, $(2K\bar{x})\bar{n}_{cmax} = 0\,2$ The constant K is generally considered to be in the order of 10^{-2} whereas the initial viscous wake width is in the order of the base radius of the body Therefore, in terms of the physical distance, the off-axis peak enthalpy reaches the axis at a distance approximately equal to 500 base radii from the initial axial station for case 3, whereas it is only 10 and 5 base radii for cases 2 and 1, respectively Since the peak enthalpy is off the axis from $\bar{x} = 0$ up to $\bar{x} = \bar{x}\bar{n}_{cm\ x}$, hence the hottest portion of the wake also spreads through this region This hottest portion of the wake is, therefore, approximately 500 base radii long for case 3, compared with 10 and 5 base radii for cases 2 and 1, respectively, for the conditions used in the present calculations Since the external inviscid conditions are assumed uniform, the electron density distribution follows the same trend as the enthalpy distribution

It is further noted that the variations of the centerline enthalpy for these three cases can be predicted qualitatively by the values of their corresponding eddy viscosity coefficients (Fig 3) In the present case, the centerline enthalpy is larger than the inviscid enthalpy Therefore, ρ is smaller than ρ and hence the value of μ_{ϵ_3} is smaller than μ_{ϵ_2} and they are in turn smaller than μ_{ϵ_1} [Eqs (1–3)] With a smaller μ_ϵ or slower diffusion process, a slower axial change of a [Eq (10)] and hence those of $\bar{\rho}\bar{u}$, $\bar{u}$, and $\bar{h}$ result This is clearly indicated in Figs 2 and 3

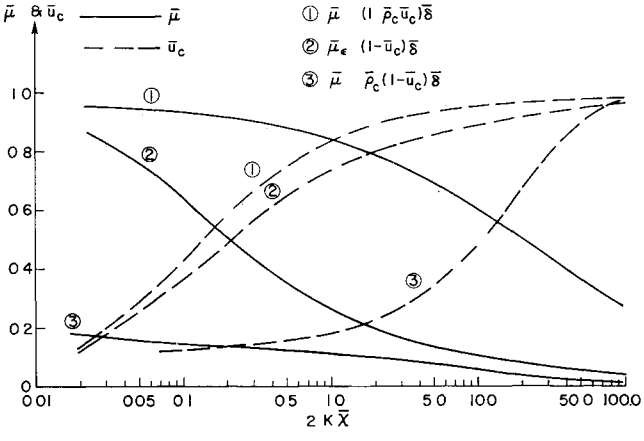

Fig 3 Variations of axis velocity and eddy viscosity coefficient; slender cone, altitude = 150,000 ft, M_∞ = 19 52

The forementioned reasoning generally applies to a slender cone under hypersonic flight conditions Therefore, the significant differences in the equilibrium turbulent-wake characteristics due to the different forms of the eddy viscosity coefficient as given here may be considered as typical for a hypersonic slender cone

Since all the three forms of the eddy viscosity coefficients considered here reduce to the same form in incompressible case but give different results for the wake characteristics, firmer ground derived from experimental evidence on which the selection of the form of the eddy viscosity coefficient can be based is much needed †

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† After this work had been completed, Ref 7 came to the author's attention The same problem treated here is also considered in that work with a finite difference method of solution, using the exact expressions given in Refs 2–4 and 8 The conclusions agree qualitatively with the present work

Correlation of Boost Phase Turbulent Heating Flight Data

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Nomenclature

c_p = specific heat at constant pressure
G = axisymmetric factor defined by Eq (3)
h = convective heat transfer coefficient
k = coefficient of thermal conductivity
Nu^* = Nusselt number, hs/k^*
P = pressure
Pr^* = Prandtl number, $\mu^* c_p^*/k^*$
r = radius of surface point from axis of symmetry
R_b = nose radius of curvature
Re_s^* = Reynolds number, $\rho^* u\, s/\mu^*$
s = boundary layer length
u = velocity component parallel to surface
γ = specific heat ratio
μ = dynamic viscosity
ρ = density

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Subscripts

e = evaluated at local edge of boundary-layer conditions
∞ = evaluated at freestream conditions
t = evaluated at local stagnation conditions

Superscript

$*$ = evaluated at Eckert reference temperature

Introduction

THE problem of predicting turbulent boundary-layer aerodynamic heating to unyawed bodies of revolution has received considerable attention in recent years The prediction of aerodynamic heat-transfer rates depends upon a knowledge of the detailed boundary-layer characteristics Solutions of the boundary-layer conservation equations often can be obtained with a high degree of precision for laminar flows, whereas turbulent boundary-layer solutions are less exact and involve the use of empirical laws The purpose of this note is to illustrate the correlation between recent flight data and two suggested methods of predicting turbulent boundary-layer aerodynamic heat transfer

The flight data reported herein were obtained from a satellite vehicle at freestream Mach numbers ranging from 1 41–4 20, and freestream Reynolds numbers ranging from 3 02 $\times$ 10^6/ft to 2 62 $\times$ 10^5/ft The 1959 Air Research and Development Command atmosphere was utilized throughout the calculations presented herein Figure 1 illustrates the vehicle configuration and variation of freestream trajectory parameters Instrumentation consisted of silicon semiconductors which were located on the cylindrical section between 5 and 27 in from the beginning of the cylindrical section as shown on Fig 1 Preflight measurements of skin thickness (nominal 0 071 in magnesium) and pictures of the installation were obtained to aid in the data reduction A surface emissivity of 0 75 consistent with the surface specifications was used for the calculation of the radiation heat transfer and was assumed to be constant A postflight analysis was conducted to determine the vehicle pitch and yaw time histories The results of this study indicated that the maximum variation of the total angle of attack in the time interval for which data are reported was less than $\pm$0 5°

Data Reduction

The inviscid flow field was obtained from a combination of the Swigart[1] blunt-body solution and a method of characteristics solution proposed by Benson [2] The Swigart solution,

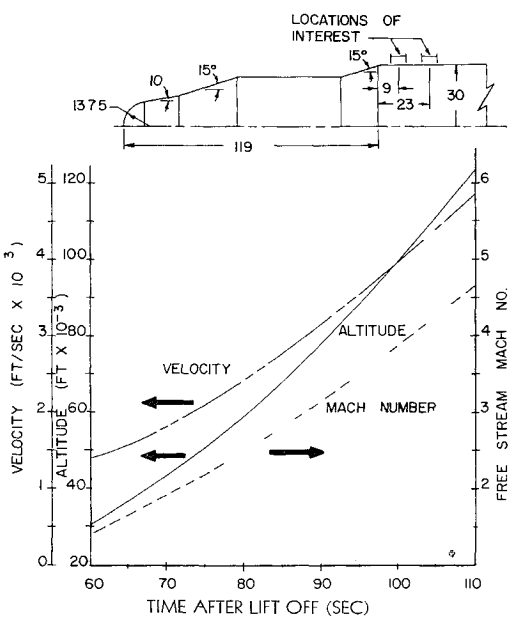

Fig 1 Freestream trajectory parameters

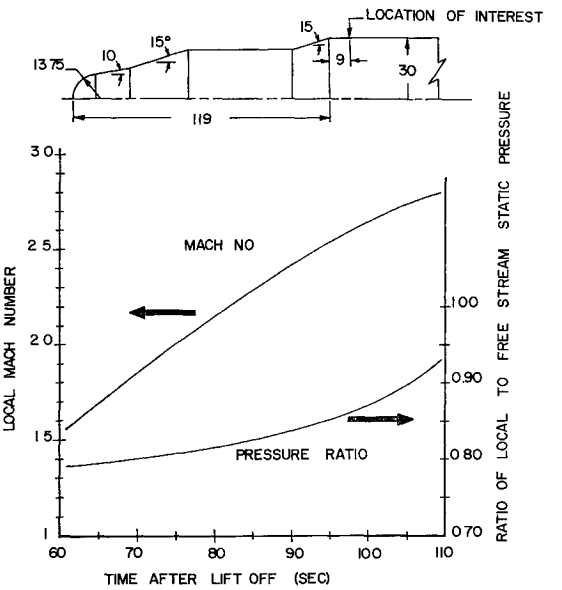

Fig 2 Variation of local flow-field parameters

which solves for the flow in the subsonic and transonic regions behind the bow shock, is used to determine the starting line for the method of characteristics solution A detailed description of the flow-field calculation procedure is presented by Lanfranco [3] Figure 2 presents the variation with time of the local Mach number and static pressure ratio for the section located between 5 and 12 in from the flare-cylinder junction

A simple thin-skin analysis which considered both convection and external radiation was used to calculate the surface heat fluxes The thermal model assumed that energy exchange could occur only at the external surface During the time interval for which data are reported, internal radiation and substructure conduction were negligible and thus were neglected In order to simplify the presentation and to average data scatter, the convective heat rates calculated for the instrumented sections (4 sensors in each location) located between 5 and 12 in and 21 and 27 in from the flare-cylinder junction were arithmetically averaged and considered to represent the convective heat rates at the midpoint of the respective section Prior to 90 sec after lift-off, the average values of the heat rates were computed at 2-sec intervals Between 90 and 103 sec the averages were computed at 1-sec intervals Reduction of the data was performed on an IBM 7090 digital computer Use was made of the averaged heat rates to calculate the corresponding heat-transfer coefficients These heat-transfer coefficients were used for the calculation of the Nusselt numbers, which are presented in Fig 3 as a function of local Reynolds number with properties evaluated at the Eckert[4] reference temperature

Data Correlation

Two methods of predicting turbulent boundary-layer convective heat transfer are compared to the flight data The first technique is that suggested by Bromberg[5] et al and modified by Hearne[6] et al for calculating convective heat transfer to a body of revolution Application of this technique yields the following expression for the Nusselt number,

$$Nu^* = 0\ 0289\ (Pr^*)^{0.333}(Re_s^*)^{0.8}G(s/R_b) \qquad (1)$$

where

$$G(s/R_b) = \left[\frac{(\rho_e/\rho_\infty)(u/u_\infty)(\mu_e/\mu_\infty)^{0.25}(r/R_b)^{1.25}(s/R_b)}{\int_0^{s/R_b}(\rho/\rho_\infty)(u/u_\infty)(\mu/\mu_\infty)^{0.25}(r/R_b)^{1.25}d(s/R_b)}\right]^{0.20} \qquad (2)$$

By successive application of isentropic flow relations along the stagnation point streamline in the region bounded by the stagnation point and the conical shock, the region bounded by the conical shock and the flare shock, and the region bounded by the flare shock and the point of interest, one can reduce Eq (2) to the following form:

$$G\left(\frac{s}{R_b}\right) = \left[\frac{(P_e/P_t)^{(5\gamma+19)/24\gamma_e}\,[1-(P_e/P_t)^{(\gamma-1)/\gamma}]^{0.5}\,(r/R_b)^{1.25}(s/R_b)}{\int_0^{s/Rb}(P/P_t)^{(5\gamma+19)/24\gamma_e}\,[1-(P/P_t)^{(\gamma-1)/\gamma}]^{0.5}\,(r/R_b)^{1.25}d(s/R_b)}\right]^{0.2} \tag{3}$$

A second heat-transfer calculation technique which is often used in the literature for data correlation is that suggested by Eckert [4] The corresponding Nusselt number correlation can be represented as follows:

$$Nu^* = 0.0296\,(Pr^*)^{0.333}\,(Re_s^*)^{0.8} \tag{4}$$

The two methods just outlined were utilized to obtain the correlation between predictions and data presented in Fig 3 Initially, Eqs (1) and (4) were used to predict Nusselt numbers for each of the instrumented sections However, the results were nearly the same; thus an average value is presented Early time (high Reynolds number) data are subject to inaccuracies because of the small difference between the recovery and wall temperatures At a flight time of 100 sec when the local Reynolds number is approximately 10^6, boundary-layer transition appears to commence Therefore, subsequent data cannot be compared with the turbulent predictions

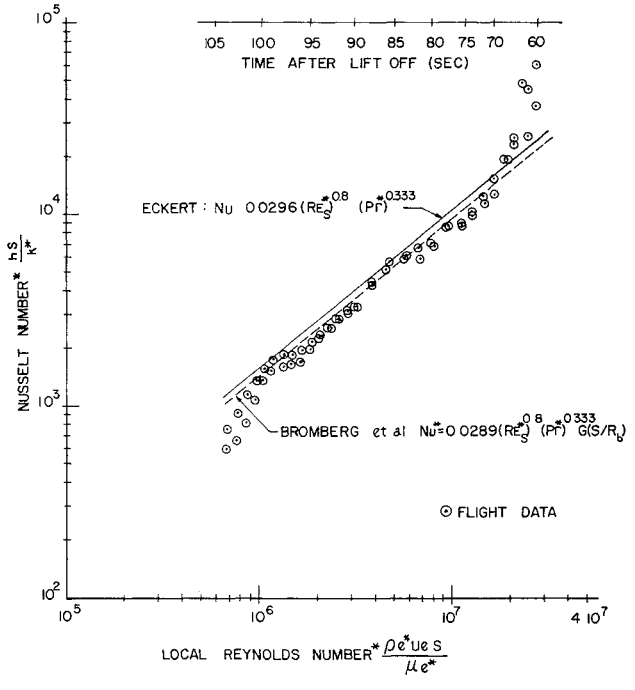

Fig 3 Comparison of observed and predicted Nusselt numbers

Conclusions

A correlation between two turbulent boundary-layer heat-transfer prediction techniques and recent flight data has been presented The Eckert correlation results in predicted Nusselt numbers which are approximately 10% greater than corresponding Bromberg values

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Transient Temperature Variation in a Thermally Orthotropic Cylindrical Shell

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An analysis is carried out to study the two-dimensional transient heat-transfer characteristics of a thin, anisotropic cylindrical shell An arbitrary heat input is assumed on the front face of the shell and the rest of the surfaces are assumed to be insulated

Giedt and Hornbaker[1] carried out an analysis to study the time-temperature history in a thermally anisotropic plate Such solutions have recently become necessary because of the increased use of such highly anisotropic materials as pyrolytic graphite for thermal protection

A similar solution applied to an infinite cylindrical shell (Fig 1) will be discussed here, subject to the same boundary conditions as those given by Giedt and Hornbaker The Fourier law of conduction for an anisotropic medium in cylindrical coordinates is given by

$$\frac{\partial v}{\partial t} = \kappa\left[\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r}\frac{\partial v_r}{\partial r}\right] + \frac{\kappa_\theta}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} \tag{1}$$

κ_r = thermal diffusivity in the r direction
κ_θ = thermal diffusivity in the θ direction
k_r = thermal conductivity in the r direction
v = temperature
t = time

Now let

$$r' = \frac{r}{(\kappa_r)^{1/2}} \qquad \theta' = \theta\left(\frac{\kappa_r}{\kappa_\theta}\right)^{1/2} \tag{1a}$$

Substitution of these new variables in Eq (1) gives

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v_r}{\partial r'^2} + \frac{1}{r'}\frac{\partial v_r}{\partial r'} + \frac{1}{r'^2}\frac{\partial^2 v_\theta'}{\partial \theta^2} \tag{2}$$

The appropriate initial and boundary conditions are

$$\frac{\partial v}{\partial r'} = 0 \text{ at } r' = \frac{a}{(\kappa)^{1/2}} = a' \qquad 0 < \theta < \theta_0 \tag{2a}$$

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